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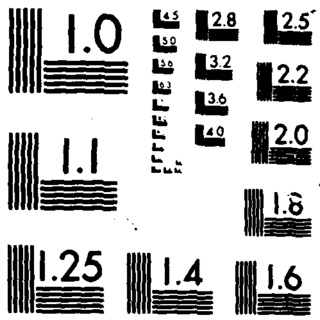

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GREEN'S FUNCTIONS OF THE REDUCED WAVE EQUATION BETWEEN PARALLEL PLANES

BY

E A SKELTON

Summary

The Green's functions in the region between planes on which either Neumann or Dirichlet boundary conditions apply are obtained in the form of double Fourier series expansions. A closed form expression for the sound field in an axisymmetric water filled channel is obtained in order to illustrate the usefulness of the expansions. Problems associated with numerical evaluation are discussed and some numerical examples are given.

Keywords: Great Britain; boundary value problems; fluid - structure interactions.

22 pages
3 figures

ARE(Teddington)
Queen's Road
TEDDINGTON Middlesex TW11 0LN

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CONTENTS

1. INTRODUCTION

The expansions of the free-space Green's functions of the reduced wave equation in Cartesian, cylindrical and spherical polar coordinate systems have been used extensively in the literature to solve boundary value problems, in fluid-structure interaction, in which at least one of the coordinate directions is infinite. One of the Green's function expansions tabulated by Skelton [1] appears to be relatively new to the literature (see equation (2.2) of this memorandum). James [2] and Skelton [3] have used it to facilitate solution of the problem of infinite pipes and layered cylinders, respectively, that are excited by an interior point source of sound: Fuller has used it to investigate infinite shell vibration [4] and the sound transmission into an idealised model of an aircraft fuselage [5]. Thus, when new forms of Green's functions become available they are invariably accompanied by the solution of a class of problems.

It is desirable to extend the range of Green's function expansions to enable solution of problems involving finite geometries. For example, the sound field inside a finite cylindrical enclosure, and the radiation of sound from the elastic vibrations of its surface are problems of considerable interest in the field of noise and vibration. Solution of these problems would be facilitated if the Green's function in the region between planes, on which Neumann or Dirichlet boundary conditions are applied, were available as double Fourier series expansions in the axial and circumferential coordinates of a cylindrical coordinate system. The Green's function expansions when the source is on the z-axis are well known from sound propagation work [6], but the extensions to an off-axis source have not been found in the literature.

2. GREEN'S FUNCTIONS

(a) General

An infinite slab of acoustic fluid is located between the planes $z=0$ and $z=h$ on which either Neumann (hard) or Dirichlet (pressure release) boundary conditions apply. A time-harmonic point source of sound of free-field pressure $\exp(ik|\underline{R}-\underline{R}_0|)/|\underline{R}-\underline{R}_0|$ is located in the acoustic fluid at the cylindrical coordinates $(r,\phi,z) = (r_0,0,z_0)$. $k=\omega/c$ is the acoustic wavenumber in which ω is the angular frequency and c is the sound velocity. The geometry is shown in Figure 1. The time-harmonic factor, $\exp(-i\omega t)$, is omitted from all equations.

The pressure in the fluid layer satisfies the inhomogeneous reduced wave equation

$$(\nabla^2+k^2)p(\underline{R},\underline{R}_0) = -4\pi\delta(\underline{R}-\underline{R}_0) \qquad (2.1)$$

together with the selected Neumann or Dirichlet boundary

conditions at $z=0$ and $z=h$. The pressure may thus be written as the sum of the free-space Green's function $p_G(\underline{R},\underline{R}_0)$ together with a scattering term $p_S(\underline{R},\underline{R}_0)$ which satisfies the homogeneous form of equation (2.1) and whose amplitude is determined by the boundary conditions.

(b) Free-Space Green's Functions

The free-space Green's function of the reduced wave equation in cylindrical polar coordinates in a form which is appropriate to the application of additional boundary conditions on cylindrical boundaries is [1]

$$p_G(\underline{R},\underline{R}_0) = \begin{cases} (i/2) \sum_{n=0}^{\infty} e_n \cos(n\phi) \int_{-\infty}^{\infty} J_n(\gamma r) H_n(\gamma r_0) \exp(i\alpha[z-z_0]) d\alpha & r \leqslant r_0 \\ (i/2) \sum_{n=0}^{\infty} e_n \cos(n\phi) \int_{-\infty}^{\infty} J_n(\gamma r_0) H_n(\gamma r) \exp(i\alpha[z-z_0]) d\alpha & r \geqslant r_0 \end{cases} \quad (2.2)$$

where J_n and $H_n=J_n+iY_n$ are Bessel and Hankel functions, respectively; and $\gamma=(k^2-\alpha^2)^{1/2}$, with $Im(\gamma)\geqslant 0$ in order to satisfy the radiation condition.

The infinite series in equations (2.2) converge only in a generalised function sense. They converge numerically provided that dissipation is included in the system by setting, for example, the sound velocity c to be the complex value $c(1-i\eta)$ where η is a very small hysteretic loss factor. However, when $r=r_0$ the series cannot be evaluated numerically even when dissipation is included.

An alternative form of equation (2.2) which is required later is determined from an application of a variant of Graf's addition theorem for Bessel functions [7], viz.,

$$H_0(\gamma\sqrt{[r^2+r_0^2-2rr_0\cos\phi]}) = \sum_{n=0}^{\infty} e_n H_n(\gamma r_0) J_n(\gamma r) \cos(n\phi) \qquad r_0 \geqslant r \quad (2.3)$$

in which, if $r_0 < r$ it is necessary only to interchange r_0 and r. The required form is obtained, by substituting equation (2.3) into equation (2.2), as

$$p_G(\underline{R},\underline{R}_0) = (i/2) \int_{-\infty}^{\infty} H_0(\gamma\sqrt{[r^2+r_0^2-2rr_0\cos\phi]}) \exp(i\alpha[z-z_0]) d\alpha \quad (2.4)$$

(c) Application of the Method of Images

It is well known [6] that the scattered pressure due to the presence of a single rigid plane boundary is equivalent to the pressure of a second, or 'image', point source which is located at an equal distance on the opposite side of the boundary to the original source, both sources having equal magnitude and phase. Similarly, the pressure due to the presence of a single pressure release plane boundary is equivalent to the pressure of an image source of equal magnitude to, but out of phase with the original source. For the case to be considered here in which the source is located between two parallel plane boundaries the pressure in the fluid may be calculated by summing an infinite series of image sources obtained as a result of successive mirror reflections of the source in the boundaries. The locations of the image sources (see Figure 1) on the vertical axis are straightforwardly obtained as the four groups of coordinates

$$z = 2(m+1)h+z_0$$

$$z = 2(m+1)h-z_0$$

$$z = -2mh+z_0$$

$$z = -2mh-z_0, \qquad m=0,1,2,...$$

The three combinations of boundary conditions to be considered are (i) both upper and lower boundaries are rigid, (ii) both upper and lower boundaries are pressure release, and (iii) the upper boundary is pressure release and the lower boundary is rigid.

(d) Both Boundaries are Rigid

When both of the plane boundaries are rigid it is clear that all the image sources have the same phase as the real source. The radial and azimuthal coordinates of the image sources are also identical to those of the real source. In order to calculate the Green's function in this case it is sufficient therefore to replace the $\exp(-i\alpha z_0)$ term in equations (2.2) and (2.4) by the infinite sum $I(\alpha,z_0)$, where

$$I(\alpha,z_0) = \sum_{m=0}^{\infty} \{\exp(-i\alpha[z_0-2mh]) + \exp(-i\alpha[-z_0-2mh])$$

$$+ \exp(-i\alpha[2(m+1)h+z_0]) + \exp(-i\alpha[2(m+1)h-z_0])\} \qquad (2.5)$$

This expression may be simplified to give

$$I(\alpha,z_0) = 2\cos(\alpha z_0)\sum_{m=-\infty}^{\infty}\exp(2i\alpha mh) \tag{2.6}$$

The generalised function identity obtained from the Poisson summation formula [8]

$$\sum_{m=-\infty}^{\infty}\exp(2\pi im\alpha/d) = d\sum_{m=-\infty}^{\infty}\delta(\alpha-md) \tag{2.7}$$

which is valid for integration purposes, allows equation (2.6) to be rewritten more usefully as

$$I(\alpha,z_0) = (2\pi/h)\cos(\alpha z_0)\sum_{m=-\infty}^{\infty}\delta(\alpha-m\pi/h) \tag{2.8}$$

When the $\exp(-i\alpha z_0)$ term in equations (2.2) is replaced by this expression for $I(\alpha,z_0)$ the α-integration is straightforward and the Green's function of the reduced wave equation between two parallel rigid planes takes the required form

$$p_G(\underline{R},\underline{R}_0) = \begin{cases} (i\pi/h)\sum_{n=0}^{\infty}e_n\cos n\phi\sum_{m=-\infty}^{\infty}J_n(\gamma_m r)H_n(\gamma_m r_0)\cos(m\pi z_0/h)\exp(im\pi z/h) & r\leqslant r_0 \\ (i\pi/h)\sum_{n=0}^{\infty}e_n\cos n\phi\sum_{m=-\infty}^{\infty}J_n(\gamma_m r_0)H_n(\gamma_m r)\cos(m\pi z_0/h)\exp(im\pi z/h) & r\geqslant r_0 \end{cases} \tag{2.9}$$

in which $\gamma_m=(k^2-m^2\pi^2/h^2)^{1/2}$. This may be simplified to

$$p_G(\underline{R},\underline{R}_0) = \begin{cases} (i\pi/h)\sum_{n=0}^{\infty}e_n\cos n\phi\sum_{m=0}^{\infty}e_m J_n(\gamma_m r)H_n(\gamma_m r_0)\cos(m\pi z_0/h)\cos(m\pi z/h) & r\leqslant r_0 \\ (i\pi/h)\sum_{n=0}^{\infty}e_n\cos n\phi\sum_{m=0}^{\infty}e_m J_n(\gamma_m r_0)H_n(\gamma_m r)\cos(m\pi z_0/h)\cos(m\pi z/h) & r\geqslant r_0 \end{cases} \tag{2.10}$$

Similarly, replacing the $\exp(-i\alpha z_0)$ term in equation (2.4) by the expression for $I(\alpha,z_0)$ in equation (2.8), integrating with respect to α and rearranging the resulting infinite sum gives

an alternative form of the Green's function of the reduced wave equation between two parallel rigid planes, viz.,

$$p_G(\underline{R},\underline{R}_o) = (i\pi/h)\sum_{m=0}^{\infty} e_m H_o(\gamma_m\sqrt{[r^2+r_o^2-2rr_o\cos\phi]})\cos(m\pi z_o/h)\cos(m\pi z/h) \tag{2.11}$$

(e) Both Boundaries are Pressure Release

When both of the plane boundaries are pressure release it is clear that a phase change occurs at each reflection. In order to calculate the Green's function it is again sufficient to replace the $\exp(-i\alpha z_o)$ term in equations (2.2) and (2.4) by an infinite sum $I(\alpha,z_o)$ which, in this case, is

$$I(\alpha,z_o) = \sum_{m=0}^{\infty} \{\exp(-i\alpha[z_o-2mh]) - \exp(-i\alpha[-z_o-2mh])$$

$$+ \exp(-i\alpha[2(m+1)h+z_o]) - \exp(-i\alpha[2(m+1)h-z_o])\} \tag{2.12}$$

As in the previous case this may be simplified considerably, viz.,

$$I(\alpha,z_o) = -2i\sin(\alpha z_o)\sum_{m=-\infty}^{\infty}\exp(2i\alpha mh) \tag{2.13}$$

which, after applying the generalised function identity, equation (2.7), may be written as

$$I(\alpha,z_o) = -(2i\pi/h)\sin(\alpha z_o)\sum_{m=-\infty}^{\infty}\delta(\alpha-m\pi/h) \tag{2.14}$$

When the $\exp(-i\alpha z_o)$ term in equations (2.2) is replaced by this expression for $I(\alpha,z_o)$ the α-integration is again straightforward, and the Green's function of the reduced wave equation between two parallel pressure release planes is

$$p_G(\underline{R},\underline{R}_o) = \begin{cases} (\pi/h)\sum_{n=0}^{\infty} e_n\cos n\phi\sum_{m=-\infty}^{\infty} J_n(\gamma_m r)H_n(\gamma_m r_o)\sin(m\pi z_o/h)\exp(im\pi z/h) & r \leqslant r_o \\ (\pi/h)\sum_{n=0}^{\infty} e_n\cos n\phi\sum_{m=-\infty}^{\infty} J_n(\gamma_m r_o)H_n(\gamma_m r)\sin(m\pi z_o/h)\exp(im\pi z/h) & r \geqslant r_o \end{cases} \tag{2.15}$$

in which $\gamma_m=(k^2-m^2\pi^2/h^2)^{1/2}$. This may be rewritten as

$$p_G(\underline{R},\underline{R}_o) = \begin{cases} (2\pi i/h)\sum_{n=0}^{\infty} e_n \cos n\phi \sum_{m=1}^{\infty} J_n(\gamma_m r) H_n(\gamma_m r_o) \sin(m\pi z_o/h)\sin(m\pi z/h) & r \leqslant r_o \\ (2\pi i/h)\sum_{n=0}^{\infty} e_n \cos n\phi \sum_{m=1}^{\infty} J_n(\gamma_m r_o) H_n(\gamma_m r) \sin(m\pi z_o/h)\sin(m\pi z/h) & r \geqslant r_o \end{cases} \tag{2.16}$$

Similarly, replacing the $\exp(-i\alpha z_o)$ term in equation (2.4) by the expression for $I(\alpha,z_o)$ in equation (2.14), integrating with respect to α and rearranging the resulting infinite sum gives an alternative form of the Green's function of the reduced wave equation between two parallel pressure release planes, viz.,

$$p_G(\underline{R},\underline{R}_o) = (2\pi i/h)\sum_{m=0}^{\infty} H_o(\gamma_m \surd[r^2+r_o^2-2rr_o\cos\phi])\sin(m\pi z_o/h)\sin(m\pi z/h) \tag{2.17}$$

(f) <u>One Rigid Boundary and One Pressure Release Boundary</u>

The case in which the boundary at $z=0$ is rigid and the boundary at $z=h$ is pressure release will now be considered. When one boundary is rigid and the other is pressure release, phase changes occur only at reflections at the pressure release boundary. In order to calculate the Green's function it is again sufficient to replace the $\exp(-i\alpha z_o)$ term in equations (2.2) and (2.4) by an infinite sum $I(\alpha,z_o)$ which, in this case, is

$$I(\alpha,z_o) = \sum_{m=0}^{\infty}(-1)^m\{\exp(-i\alpha[z_o-2mh]) + \exp(-i\alpha[-z_o-2mh]) - \exp(-i\alpha[2(m+1)h+z_o]) - \exp(-i\alpha[2(m+1)h-z_o])\} \tag{2.18}$$

As in the previous cases this may be simplified considerably, viz.,

$$I(\alpha,z_o) = 2\cos(\alpha z_o)\sum_{m=-\infty}^{\infty}(-1)^m\exp(2i\alpha mh) \tag{2.19}$$

This is, however, a form which is not suitable for a direct application of the generalised function identity, and it is necessary to rewrite equation (2.19) as

$$I(\alpha,z_0) = 2\cos(\alpha z_0)\{2\sum_{m=-\infty}^{\infty}\exp(4i\alpha mh) - \sum_{m=-\infty}^{\infty}\exp(2i\alpha mh)\} \qquad (2.20)$$

The generalised function identity may now be applied to each of the infinite sums in this equation, allowing it to be written as

$$I(\alpha,z_0) = (2\pi/h)\cos(\alpha z_0)\{\sum_{m=-\infty}^{\infty}\delta(\alpha-m\pi/2h) - \sum_{m=-\infty}^{\infty}\delta(\alpha-m\pi/h)\} \qquad (2.21)$$

which may itself be simplified to

$$I(\alpha,z_0) = (2\pi/h)\cos(\alpha z_0)\sum_{m=-\infty}^{\infty}\delta(\alpha-(2m+1)\pi/2h) \qquad (2.22)$$

When the $\exp(-i\alpha z_0)$ term in equations (2.2) is replaced by this expression for $I(\alpha,z_0)$ the α-integration is straightforward and the Green's function of the reduced wave equation between a pressure release plane and a parallel rigid plane is

$$p_G(\underline{R},\underline{R}_0) = \begin{cases} (i\pi/h)\sum_{n=0}^{\infty}e_n\cos n\phi\sum_{m=-\infty}^{\infty}J_n(\gamma_m r)H_n(\gamma_m r_0)\cos(\alpha_m z_0)\exp(i\alpha_m z) & r\leqslant r_0 \\ (i\pi/h)\sum_{n=0}^{\infty}e_n\cos n\phi\sum_{m=-\infty}^{\infty}J_n(\gamma_m r_0)H_n(\gamma_m r)\cos(\alpha_m z_0)\exp(i\alpha_m z) & r\geqslant r_0 \end{cases}$$

(2.23)

in which $\alpha_m=(2m+1)\pi/2h$, and $\gamma_m=(k^2-\alpha_m^2)^{1/2}$. This may be re-written as

$$p_G(\underline{R},\underline{R}_0) = \begin{cases} (2i\pi/h)\sum_{n=0}^{\infty}e_n\cos n\phi\sum_{m=0}^{\infty}J_n(\gamma_m r)H_n(\gamma_m r_0)\cos(\alpha_m z_0)\cos(\alpha_m z) & r\leqslant r_0 \\ (2i\pi/h)\sum_{n=0}^{\infty}e_n\cos n\phi\sum_{m=0}^{\infty}J_n(\gamma_m r_0)H_n(\gamma_m r)\cos(\alpha_m z_0)\cos(\alpha_m z) & r\geqslant r_0 \end{cases}$$

(2.24)

Similarly, replacing the $\exp(-i\alpha z_0)$ term in equation (2.4) by the expression for $I(\alpha,z_0)$ in equation (2.22), integrating with respect to α and rearranging the resulting infinite sum gives an alternative form of this Green's function, viz.,

$$p_G(\underline{R},\underline{R}_0) = (2i\pi/h)\sum_{m=0}^{\infty} H_0(\gamma_m\surd[r^2+r_0^2-2rr_0\cos\phi])\cos(\alpha_m z_0)\cos(\alpha_m z) \tag{2.25}$$

3. SOUND FIELD IN CHANNEL

(a) Mathematics

The use of the Green's functions derived in Section 2 is illustrated by considering the problem of finding the pressure due to a time-harmonic source which is located in an axisymmetric channel containing water. Figure 2 shows the geometry. On the free surface at $z=h$ a Dirichlet boundary condition applies, and on the other boundaries defined by the surfaces $z=0$, $r=a$ and $r=b$, Neumann boundary conditions apply. The pressure in the channel may be written as the sum of the Green's function of Section 2(f), together with a scattering term which satisfies the homogeneous form of the wave equation (2.1) and the boundary conditions on the planes $z=0$ and $z=h$. Thus

$$p(r,\phi,z) = p_G(r,\phi,z) + p_S(r,\phi,z) \tag{3.1}$$

in which $p_G(r,\phi,z)$ is given by equation (2.24), and the scattering term is given by

$$p_S(r,\phi,z) = \sum_{n=0}^{\infty} e_n\cos n\phi \sum_{m=0}^{\infty} \{A_{mn}J_n(\gamma_m r)+B_{mn}H_n(\gamma_m r)\}\cos(\alpha_m z) \tag{3.2}$$

The unknown coefficients A_{mn} and B_{mn} may be determined by imposing the Neumann boundary conditions $\partial p(r,\phi,z)/\partial r=0$ on the cylindrical surfaces $r=a$ and $r=b$, to give the matrix relation

$$\underset{2*1}{\begin{bmatrix} A_{mn} \\ B_{mn} \end{bmatrix}} = (-2\pi i\cos(\alpha_m z_0)/hD_{mn}) \underset{2*2}{\begin{bmatrix} H_n'(\gamma_m b) & -H_n'(\gamma_m a) \\ -J_n'(\gamma_m b) & J_n'(\gamma_m a) \end{bmatrix}} \underset{2*1}{\begin{bmatrix} J_n'(\gamma_m a)H_n(\gamma_m r_0) \\ J_n(\gamma_m r_0)H_n'(\gamma_m b) \end{bmatrix}} \tag{3.3}$$

where

$$D_{mn} = J_n'(\gamma_m a)H_n'(\gamma_m b)-H_n'(\gamma_m a)J_n'(\gamma_m b) \tag{3.4}$$

is the dispersion relation whose zeros $\omega=\omega_{mn}$ are the natural frequencies of the system.

From a computational point of view it is shown below that the alternative form of the Green's function given by equation (2.25) is better suited to the numerical evaluation of equation (3.1).

(b) Convergence of Series

In order to investigate the convergence of the series in equations (2.24) and (3.2) it is necessary to consider the behaviour of the terms as $m \to \infty$. In this case γ_m is a monotonically increasing positive imaginary number ix_m, say. The asymptotic forms of the Bessel functions

$$J_n(ix_m) \sim \exp[x_m + i(n\pi/2+\pi/4)]/(2i\pi x_m)^{1/2}$$
$$H_n(ix_m) \sim 2\exp[-x_m - i(n\pi/2+\pi/4)]/(2i\pi x_m)^{1/2} \tag{3.5}$$

are then used to obtain the asymptotic forms

$$A_{mn}J_n(\gamma_m r) \propto \exp[x_m(r-b)+x_m(r_o-b)] \tag{3.6}$$

$$B_{mn}H_n(\gamma_m r) \propto \exp[x_m(a-r)+x_m(a-r_o)] \tag{3.7}$$

of the coefficients in the series expansions of the scattered pressure, $p_S(r,\phi,z)$. Clearly, unless $r=r_o=b$, equation (3.6) decreases exponentially as $x_m \to \infty$. Additionally, unless $r=r_o=a$, equation (3.7) decreases exponentially as $x_m \to \infty$. Thus, unless both source and receiver are situated on the inner or the outer cylindrical boundary, rapid convergence of the scattered pressure summation is assured.

If the same reasoning is applied to the terms in the Green's function expansion given by equation (2.24) it becomes evident that

$$J_n(\gamma_m r)H_n(\gamma_m r_o) \propto \exp[x_m(r-r_o)], \qquad r \leqslant r_o \tag{3.8}$$

$$J_n(\gamma_m r_o)H_n(\gamma_m r) \propto \exp[x_m(r_o-r)], \qquad r \geqslant r_o \tag{3.9}$$

Thus unless $r=r_o$ convergence is assured but at a rate which is less than that of the scattered term. When $r=r_o$ the series does not converge numerically - it exists only as a generalised function. The terms in the alternative form of the Green's function decay exponentially as $x_m \to \infty$ unless both $r=r_o$ and $\phi=0$. Thus the alternative form of the Green's function is suitable for numerical evaluation everywhere except when the receiver is on a vertical line through the source.

The results of this Section are applicable to other problems of this kind. The main point that arises is that the alternative forms of the Green's functions should be used, where practical, when numerical computations are required.

4. NUMERICAL RESULTS

A Fortran program has been written to calculate the sound field due to a time harmonic source located in an axisymmetric channel. The first version of the program computed the Green's function using equation (2.24). Severe numerical problems were encountered. When the alternative form of the Green's function equation (2.25) was used in the computations rapid convergence was achieved, thus helping to validate the analysis of the previous Section.

In the plots of Figure 3 the following constants in SI units were used:

$h=0.5$	$c=1500.0$
$a=1.0$	$b=2.0$
$r_0=1.5$	$r=1.5$
$z_0=0.25$	$z=0.25$

Thus both source and receiver are located at midwater. The sound levels are in dB ref. 1 micropascal, and are to be referred to a free field source level of 120dB at 1m.

In Figure 3A the angular separation between source and receiver is $\phi=90°$. The sound level spectrum consists of numerical values at 8Hz spacing, which are joined by straight lines. Below the cut-on frequency of the first mode at 750Hz the sound pressure is negligible. Above this cut-on frequency the spectrum is dominated by very sharp resonant peaks whose levels would increase without bound if the resolution in frequency were increased. This is because of the absence of dissipation in the system. In Figure 3B the angular separation between source and receiver is the maximum of 180°. The feature which distinguishes this plot from the previous one is the increased density (factor of 2) of resonant peaks, simply due to the factor $\cos(n\phi)$ in the summation which vanishes for odd values of n when $\phi=90°$.

5. CONCLUDING REMARKS

The Green's functions in the region between parallel planes on which either Neumann or Dirichlet boundary conditions apply have been presented in a form which facilitates:

(i) Theoretical analysis of certain boundary value problems in finite axisymmetric regions.

(ii) Numerical evaluation of the sound fields without incurring problems of severe ill-conditioning.

A closed-form expression for the sound field in an axisymmetric channel and some numerical results have been obtained as an example. The availability of Green's functions result in their application to practical problems. For example, James [9] has used the Green's functions of equation (2.10) to investigate the

sound radiation from a simply supported elastic shell which is excited by an interior point source.

E.A. Skelton (HSO)

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REFERENCES

1. SKELTON E.A., Free-Space Green's Functions of the Reduced Wave Equation, Admiralty Marine Technology Establishment, Teddington, AMTE(N)TM82073, September 1982.

2. JAMES J.H., Sound Radiation from Fluid Filled Pipes, Admiralty Marine Technology Establishment, Teddington, AMTE(N)TM81048, September 1981.

3. SKELTON E.A., Sound Radiation from a Cylindrical Pipe Composed of Concentric Layers of Fluids and Elastic Solids, Admiralty Marine Technology Establishment, Teddington, AMTE(N)TM83007, January 1983.

4. FULLER C.R., Monopole Excitation of Vibrations of an Infinite Cylindrical Elastic Shell Filled with Fluid. J. Sound. Vib., 96(1), 1984, pages 101-110.

5. FULLER C.R., Noise Control Characteristics of Synchrophasing - An Analytical Investigation. A.I.A.A. Paper 84-2369, 9th Aeroacoustics Symposium, Williamsburg, VA., October 15-17, 1984.

6. BREKHOVSKIKH L.M., Waves in Layered Media, Second Edition, Academic Press, 1980.

7. ABRAMOWITZ M., STEGUN I., Handbook of Mathematical Functions, Dover, 1965.

8. JONES D.S., The Theory of Generalised Functions, Second Edition, Cambridge University Press, 1982.

9. JAMES J.H., An Approximation to Sound Radiation From a Simply Supported Cylindrical Shell Excited by an Interior Point Source, Admiralty Research Establishment, Teddington, AMTE(N)TM85024, March 1985.

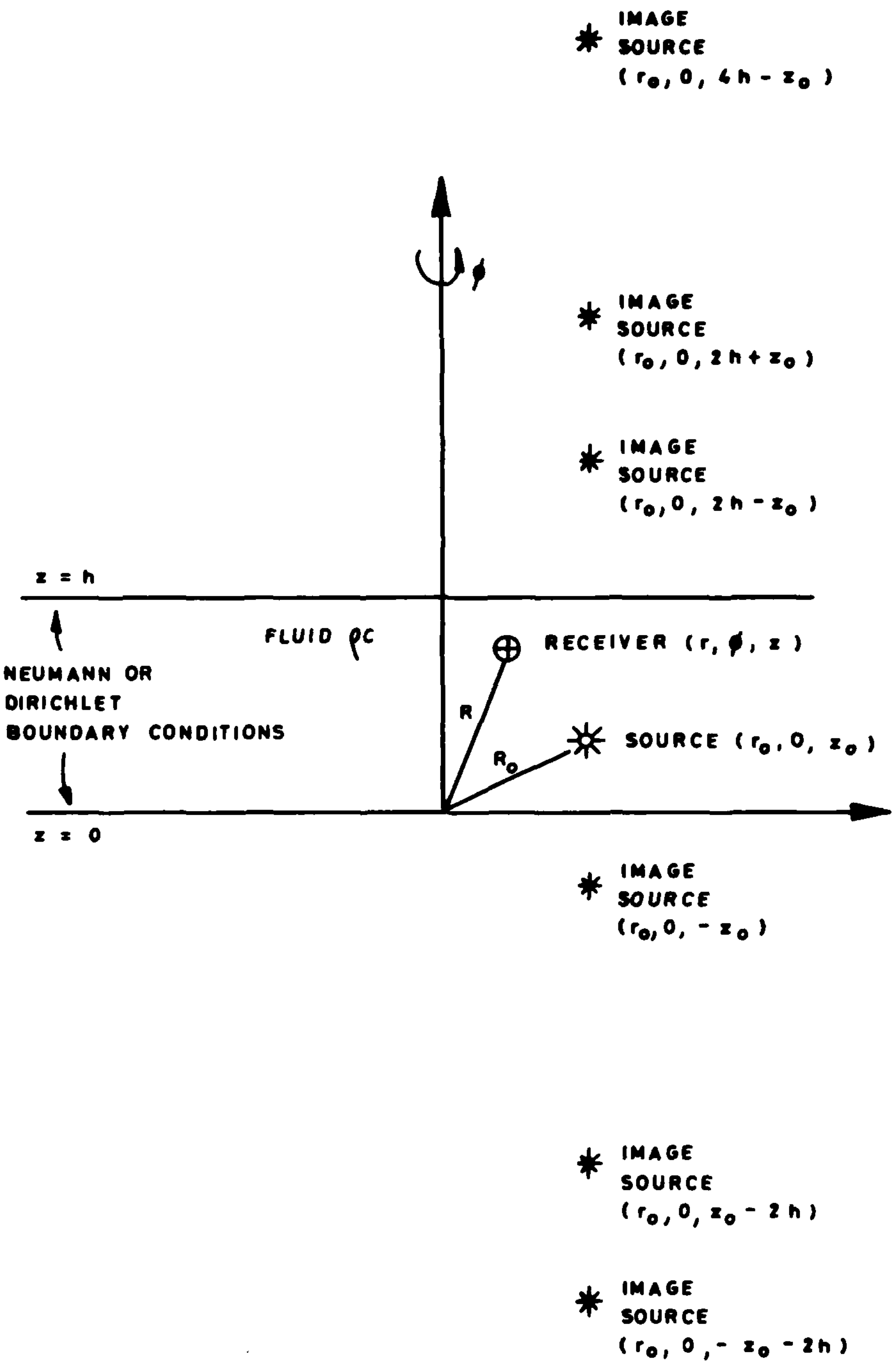


FIG. 1 GEOMETRY AND IMAGE SOURCE APPROXIMATION

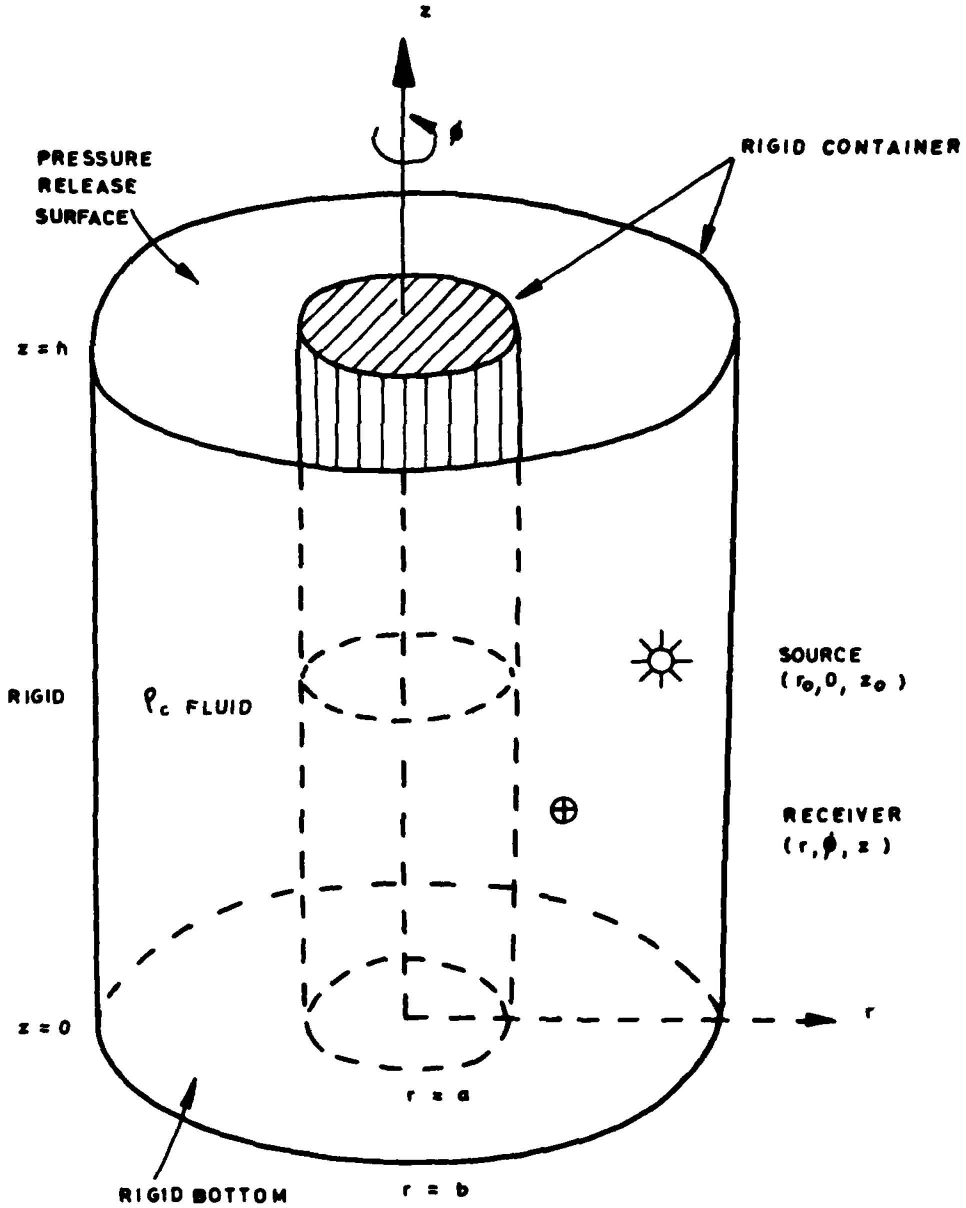
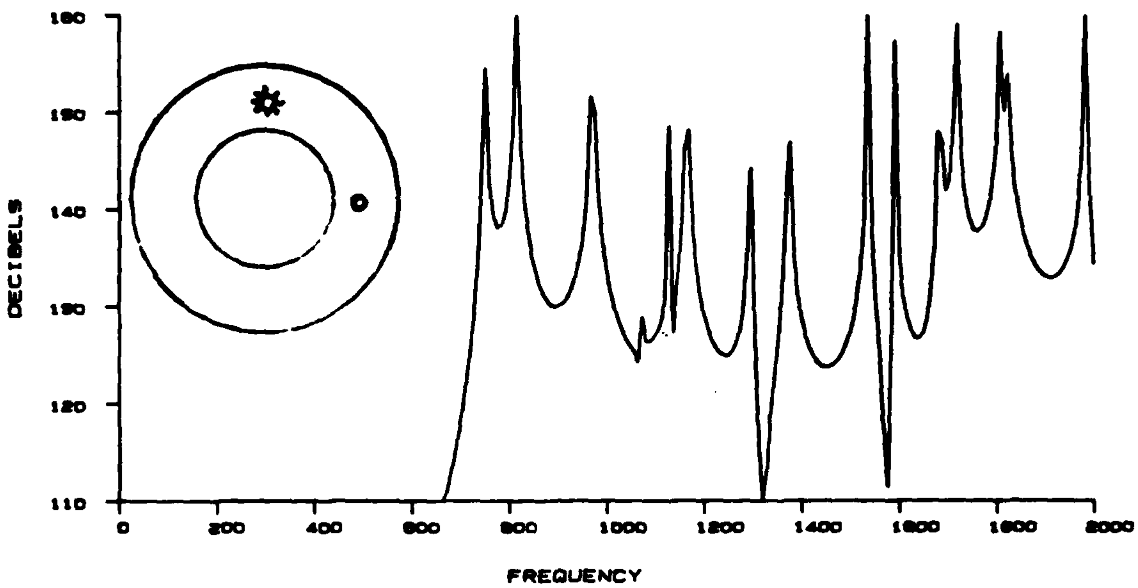


FIG. 2 GEOMETRY OF AXISYMMETRIC CHANNEL

A. SOURCE AND RECEIVER SEPARATED BY 90°



B. SOURCE AND RECEIVER SEPARATED BY 180°

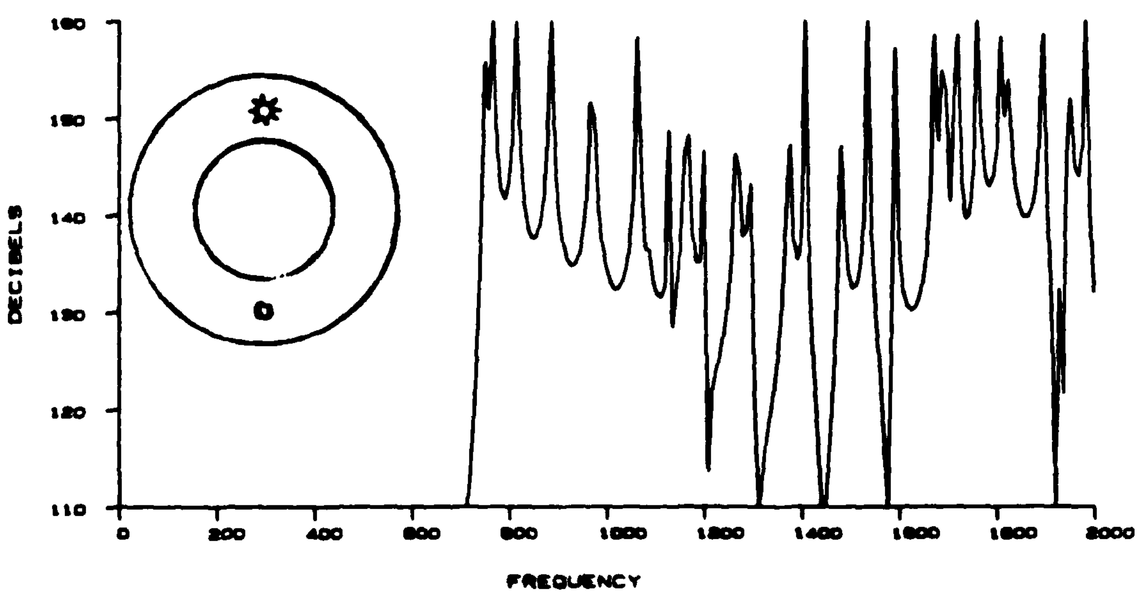


FIG. 3 SOUND FIELD IN AXISYMMETRIC CHANNEL

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